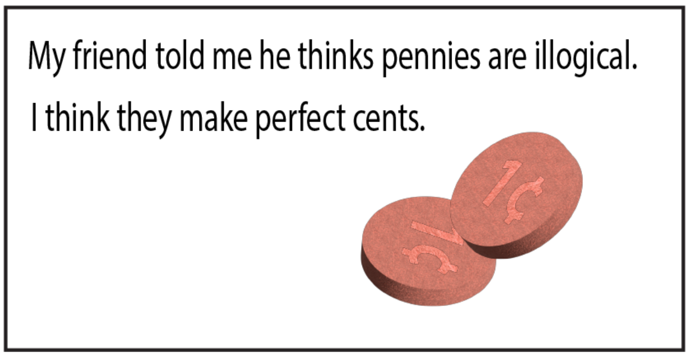
Grade 6 Math Circles
February 21/22/23, 2023
Logic Puzzles

Introduction to Logic

Logic is the concept of using critical thinking and deductive reasoning to come to a conclusion about something. This can vary from choosing to put on a coat if the Weather Network tells us that it is going to rain, to answering a comprehension question in English class.



In mathematics, we use logic all the time when working through problems. Today, though, we will focus on logic games and puzzles in order to build these skills you will need in future math courses!

Stop and Think

Is it logical to say: "I'm going to have a conversation with my dog today"? Why or why not?

Example A

Let's say that on Monday at 6pm I was watching a movie at home.

1. Was I swimming at the beach at 6pm on Monday?

   No! As humans we cannot be present in two different places at the same time, so using this reasoning I could not have been watching a movie and swimming at the same time.

2. You now have the information that I lived 1 hour away from the beach. Assuming that I drove safely and followed the speed limit, could I have been tanning by the water there at 6:30?

   No! In this question we have been given more information to allow us to answer with reason. If I was watching a movie at 6, the earliest I could leave so that the statement is still true would be 6:01. Since I live an hour away from the beach, I could only arrive there at 7:01, and so could not be there by 6:30.

Strategies

1. Look at the facts that you have been given. What do you know for certain is true? What do you know for certain is false?
2. Reread the problem a few times and look at your clues carefully: some might be giving information about not just one part of the question, but others as well!
3. Draw a picture or a diagram to organize the information that you have.
4. Try using trial and error: assume one piece of information is true, and then continue with the problem until you solve it or arrive at a contradiction.
5. Have fun and don't give up! Sometimes you need to step away from a question to see it more clearly, so don't get discouraged if you can't solve it right away.

Practice your strategies

Before we get into the nitty-gritty of logic puzzles, you will try out two famous problems to *exercise* your reasoning muscles (just like you stretch before you do any kind of physical activity, you have to stretch your brain for mental activity!)

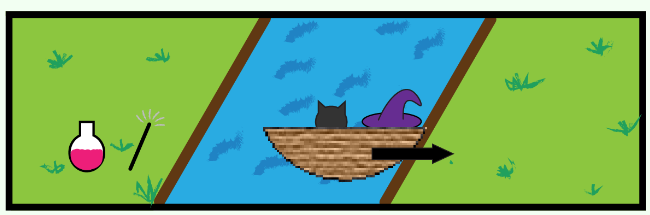
Example B

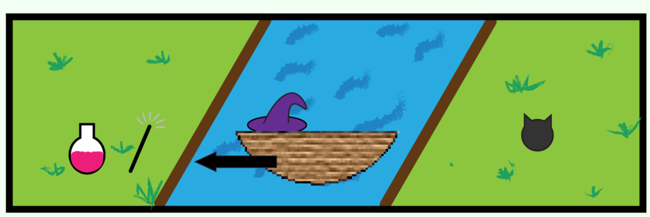
An ancient wizard has with himself a wand, a potion, and a cat. He comes to a river that he needs to cross with a boat, but can only bring with him one of the items at a time. If left alone without the wizard present, the wand would put a spell on the cat, or the cat would drink the potion. Our job is to figure out a way for the wizard to bring all of his items to the other side of the river safely. It is important to note that **only the wizard can steer the boat**.

First, let us make a list of all the information we have - both what is given, and what we can *infer* using logical reasoning:

1. The wand cannot be left alone with the cat.
2. The cat cannot be left alone with the potion.
3. The wand **can** be left alone with the potion.
4. One item at a time can stay with the wizard.

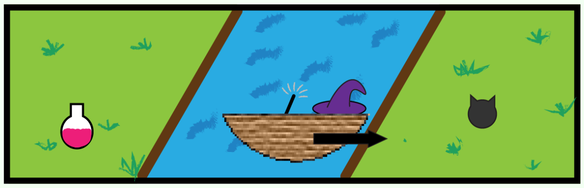
Right away we can see that the cat must be the first to travel with the wizard, as this will leave the wand alone with the potion. Once the wizard brings over the cat, he can travel back alone for the wand or the potion:

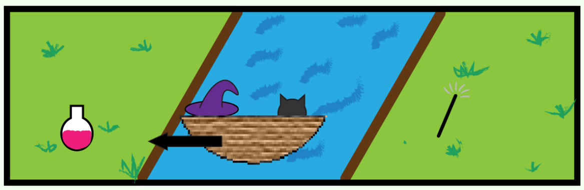




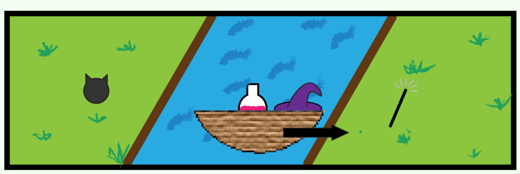
Now we have two options: to take the wand or to take the potion. At this stage, it does not matter which one we choose, and so let us go with the wand. When the wizard brings over the wand, he now has the wand and the cat together on one side. Clearly he cannot leave these two alone, so he must choose one to go back with him.

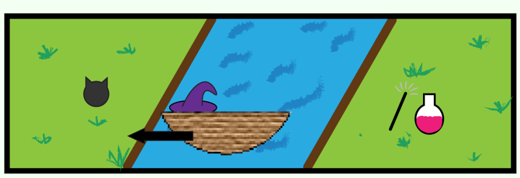
If the wizard brings back the wand, he will have just reversed the decision he made, so we choose to have him take the cat back.





Now we have the wizard, cat and the potion together on the first side, and the wand on the other side. The wizard must now travel with either the cat or the potion, since they cannot stay together. Since the wizard just brought the cat over, we choose to have him bring the potion back.
Since the potion and the wand are together on the other side, the wizard can leave them alone and go back for the cat. He can then take the cat to the other two items, and so we have now solved the problem!





The next problem is a simpler variation of **Einstein's riddle**.

Exercise 1

Four CEMC staff members were hit in the face with a pie on Pi Day. Given the following clues, determine the job, name, and type of pie that is associated with each of the staff:

1. The President of the CEMC was hit in the face with a blueberry pie.
2. CEMC's Secretary was not Nancy.
3. Dominica is either the Vice President, or slapped with an apple pie.
4. Jeffrey is the Treasurer of the CEMC and was not slapped with a pumpkin pie.
5. Nancy did not get a pumpkin nor a rhubarb pie in her face.
6. Paul was slapped with an apple pie.
7. The staff member hit with a rhubarb pie was not the Secretary.

   Hint: Use a table to organize your thoughts!

Sudoku

Sudoku is an example of a *Latin square puzzle*. The objective of these puzzles is to fill each square with a certain number such that the given conditions are satisfied. We do so by using logic and applying different strategies. Sudoku consists of a 9x9 grid that contains nine 3x3 subgrids inside of it. Each subgrid must contain the numbers 1 through 9 exactly once, and each of the rows and columns of the whole 9x9 grid must also contain the numbers 1 through 9 exactly once. Sudoku can have a range of difficulty, depending on how many numbers are initially given to you.

Square number puzzles have been around since the late 19th century, but the variation that we now recognize to be modern Sudoku appeared around the year 1979. Since gaining popularity, this puzzle has not only caught the eyes of the regular public but mathematicians as well. It has been determined that there are 6,670,903,752,021,072,936,960 different filled grids!

This puzzle is probably one of the most accessible games; you can find it in just about any store, in any format. Today we will look at the basic Sudoku puzzles as well as some variations.

Example C

Consider one row of subgrids of a Sudoku puzzle:

	4	7	5	1	9	3	6	8
1	8	5	3	2	6	4	7	9
3	9	6		4	8	2		1

The missing number in the first grid must contain the number 2, the missing number in the second grid must contain the number 7, and the missing number in the third grid must contain the number 5.

Exercise 2

Solve this easy level Sudoku Puzzle.

1	5	6		2	8		7	9
3		8	9	7	5	4	1	6
9	7	4	6	3		2		
	6	9		5	7		3	4
7	4		8		9	1	2	5
5		1	2	4	3	9	6	
8		2	5	1	6	7	4	
	3			9	2		8	1
6		7			4	5	9	

Stop and Think

What strategies did you use to solve that puzzle? What was your process?

Sudoku Strategies

1. Go subgrid by subgrid (to try and complete it) and keep shuffling through them until they are all solved.
2. Go number by number, and try to insert it into each subgrid.
3. Go row by row and/or column by column.
4. Notice patterns: For example, if there are three numbers missing, and two of those numbers both cannot be in one of the spaces, the third number must go there!

Stop and Think

Given the Sudoku row below, which square in the middle grid *must* be a 5?

	4	7		1	9	3	6	8
1	8			2	6	4		9
	9	6		4	8	2		1

As you can see, there are many ways to approach a sudoku puzzle. Depending on the difficulty, you'll need to use one strategy, two strategies, or all of them combined! Let's see if you can apply them:

Exercise 3

Solve this medium level Sudoku Puzzle.

	1	2	5	7			9	8
8		5			6			4
7	9	6	4	8	2		3	5
6				1	5	3	4	
1	5	3	2		7			9
	7	4	6		8	5	1	
3	4			5	9	2	8	6
	8		3	6		9		1
	6	9	8	2	1	4	7	

Bridges

Hashiwokakero - also known as "Bridges" - is another type of Logic puzzle. This one originated in Japan, and varies from other puzzles as it does not have any standard size. It usually contains a grid to make it easier for the solver, but it is not necessary.

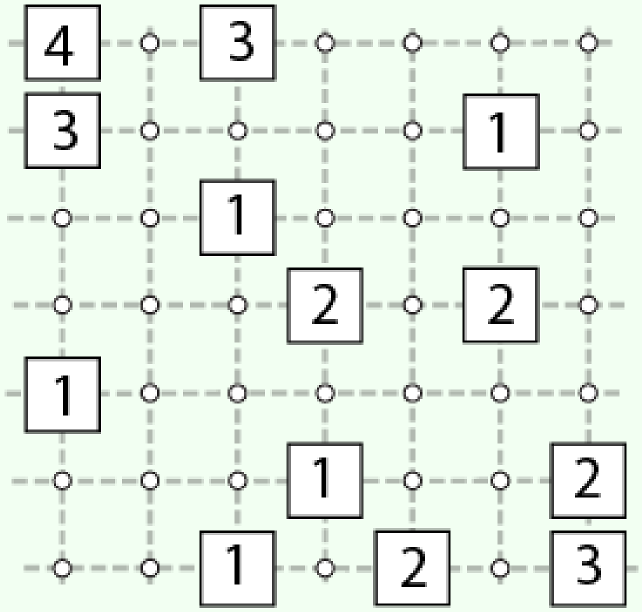
How to Play:

The objective of the game is to connect all of the islands by drawing bridges between them. The rules for drawing bridges are as follows:

1. There can be zero, one, or two bridges on each side of an island (going either horizontally or vertically)
2. Bridges cannot cross one another
3. A bridge must connect to an island on each of its ends, and they must be two different islands
4. The number of bridges connected to an island must match the number that is written on the island

Let's look at an example to see how this works:

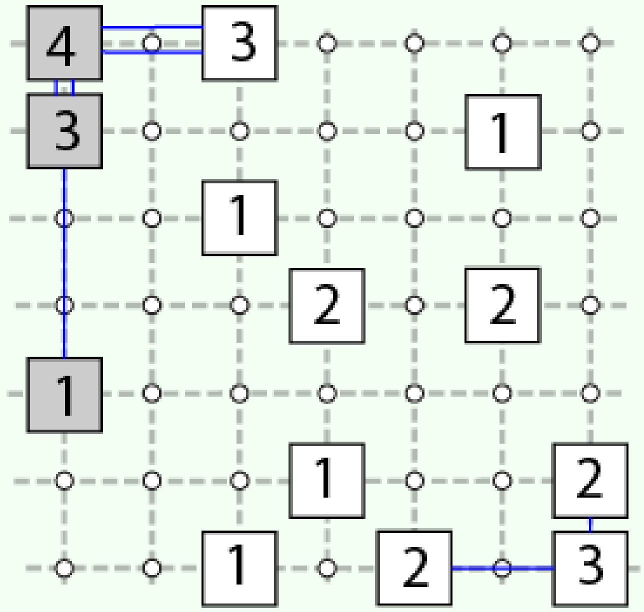
Example D



Usually the easiest place to begin is looking at any islands on the corners, as they are restricted to having bridges on only two sides instead of four. In the top left corner, we can draw all four bridges from that island: two going horizontally, and two going vertically. Next, we can draw two bridges from the bottom right island: one going horizontally and one going vertically. We can do this because there is at most two bridges on each side of an island, and this island must have three bridges. Note that we do not yet have enough information to draw the third bridge!

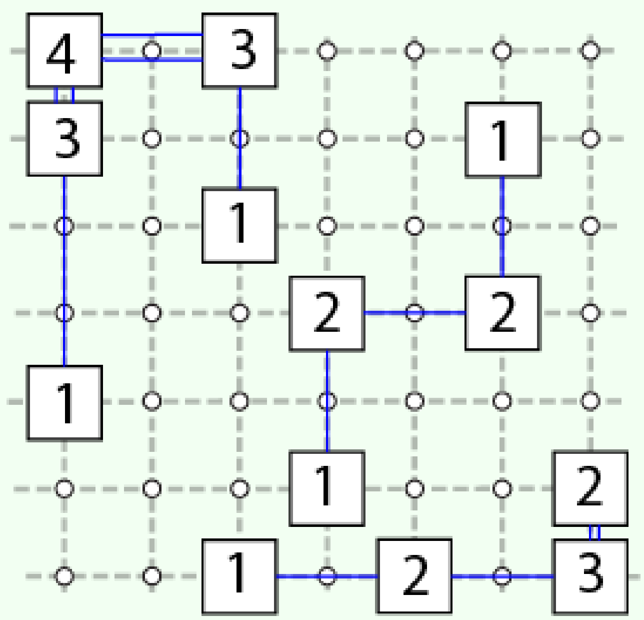
Another easy place to begin is by looking at all of the islands with the number 1, and seeing if there is exactly one island that it could connect to. For example, our island on the very left with the number 1 must have a bridge connecting vertically upwards to the island with the number 3 since there is no island to the right or below it.

You can choose to shade in the islands that have been "solved" so you know what bridges you have left to draw. Let's see what our puzzle looks like now:



Continue using deductive reasoning until you solve the puzzle. As a hint to get you started, notice that that the island with the number “1” on the top right of the grid must have a bridge connecting to the island below it, since the island to the left of it has been solved.

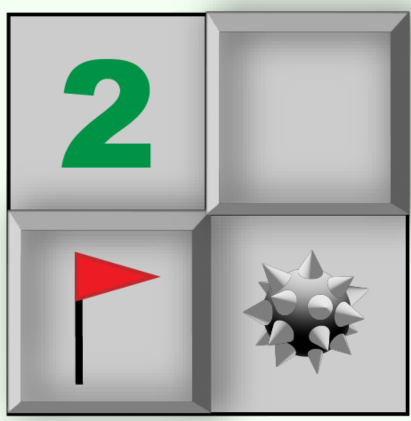
Your final grid should look like the solution below:



Minesweeper

Minesweeper is a computer game that originated between the late 1980s and early 1990s. It is another type of logic puzzle, but it varies from the others as you are going in blind right from the very beginning and making decisions based off of that very first move.

Example E



There are five ways that a square on a Minesweeper board can look - above we have four of them. The top right is an unopened square; it could have a mine underneath, or a number. On the bottom right is a mine; we do not want to see this symbol except at the end of the game. On the bottom left is an unopened square that has been flagged; this means that based on the information we have, we are fairly certain that there is a mine underneath. On the top left is a numbered square, in this case with the number 2 which means there are two mines **adjacent*** to that square.

*In this game, a square is **adjacent** to another square if their sides are touching or if their corners are touching.

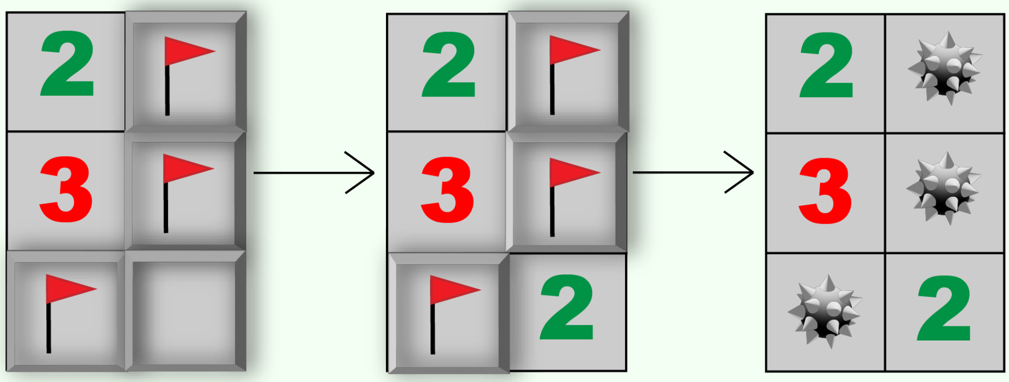
The fifth type of square on a Minesweeper board is an opened blank square, also known as the '0' square because it means there are no mines adjacent to it.

How to Play:

1. Click any square on the board.
2. Look at the surrounding squares that have been revealed, and the information they give you. They will either be blank, or have a number.
3. Place flags on the squares you are certain have a mine.
4. If you know for certain that a square does not contain a mine, click it. Then repeat steps 2-4.

   If you do not know for certain that a square does not contain a mine, and there are no other clues, click a random square and then repeat steps 2-4.

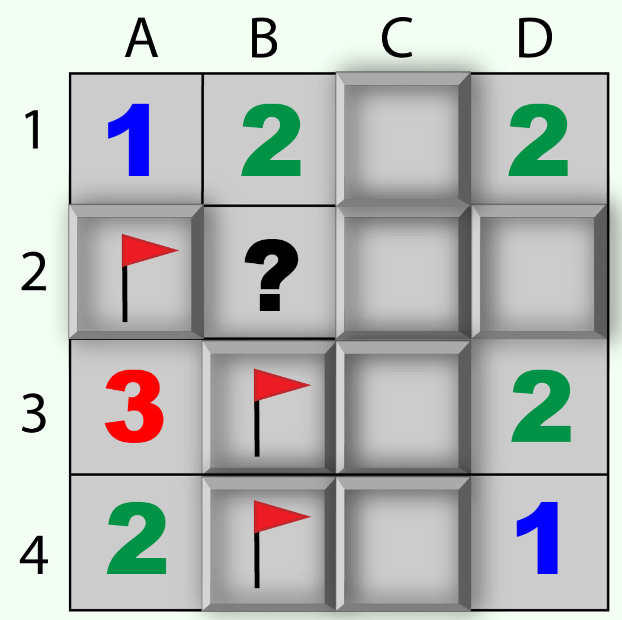
Example F



Consider the first picture. You have been given information about all the squares except the very bottom rightmost one. In the second picture it has been clicked on out of certainty that it does not have a mine, since there already three flagged squares next to the numbered '3' square. Indeed, the last picture reveals that we were correct!

Example G

Suppose you are given the following partially-filled 4x4 Minesweeper grid:

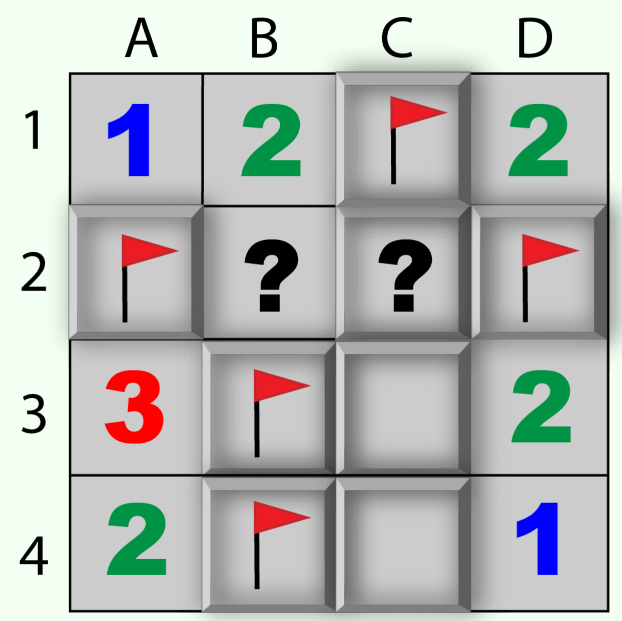


For clarity, we will refer to the boxes based on what column and row they are in. For example, the box with the number "1" in the top-left corner would be A1. The box with the number "1" in the bottom-right corner would be D4.

We now want to determine what boxes we can say for certain have mines; what boxes we can say for certain *don't* have mines; and whether there is enough information given for us to solve the whole grid. Below are the deductions we can make:

⟶ B2 cannot have a mine because if it did, then A1 would be adjacent to two mines, but the number "1" tells us it is only adjacent to one mine (on A2). So B2 must contain some number.
⟶ Both B3 and B4 have mines. This is because A3 is adjacent to four unknown boxes, one of which we have declared as "safe" (B2) thus the other three (A2, B3, B4) must have mines as per the number '3' on A3.
⟶ Either C1 has a mine or C2 has a mine, but not both. This is because B1 must be adjacent to exactly two mines; one is A2, and so the only options left for the second mine are C1 and C2 since we said B2 is "safe"

⟶ D2 must have a mine, because box D1 is adjacent to exactly two mines. We deduced above that exactly one of C1, C2 have a mine, and so the second mine has to lie in D2.
⟶ C3 or C4 must have a mine, but not both, since D4 is adjacent to exactly one mine. ⟶ C2 can't have a mine is D3 is adjacent to exactly two mines; one of these is in D2 and the other is in C3 or C4. ⟶ Since C2 is safe, C1 must have a mine.
This is all of the information we can deduce, as we would have to know the value of B2 to know anything about the mines in column C. So we have a partial solution as seen below:



Exercise 4
Try out the Easy, Medium and Hard levels of Minesweeper from Google. Once you become comfortable with solving the game, try to do it as fast as you can!

Kyudoku

Kyudoku is another Logic puzzle that consists of a 6x6 grid and uses the numbers 1 through 9. The objective of this puzzle is to circle each of the numbers 1 through 9 exactly once, such that the sum of the numbers circled in each row and column is less than or equal to 9. You are *always* given an initially circled number from which you can begin to play!

How to Play:

1. Shade in all of the boxes with the number that is initially circled. Let us call this number n.
2. Shade in all of the boxes with numbers that are greater than $9 - n$ in the row and column containing n.
3. Use different strategies to circle each of the other numbers, repeating step 2 after they are circled. We will discuss strategies below!

Example H

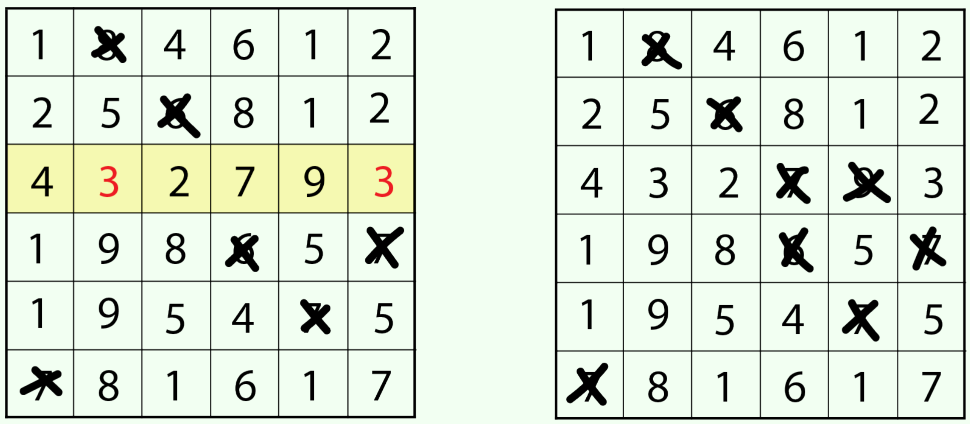
3	3	8	3	1	4
1	9	4	9	7	8
9	6	2	9	6	5
4	8	4	5	5	6
5	6	7	4	1	6
4	2	8	2	**9**	1

Solution

3	3	8	**3**	1	4
1	9	4	9	7	**8**
9	**6**	**2**	9	6	5
4	8	4	**5**	5	6
5	6	**7**	4	1	6
4	2	8	2	**9**	1

Kyudoku Strategies

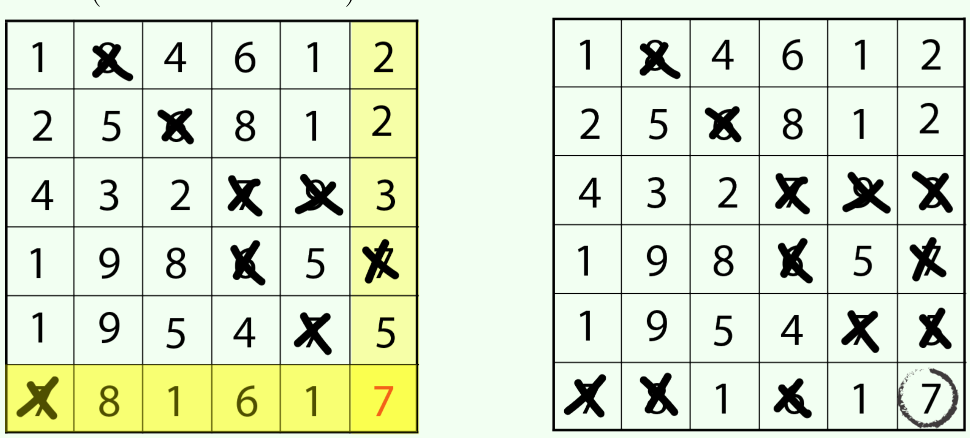
1. Try to identify numbers that only exist in one row or column. Since this row/column **must** have this number circled, you can shade in all the other numbers that are greater than $9 - n$



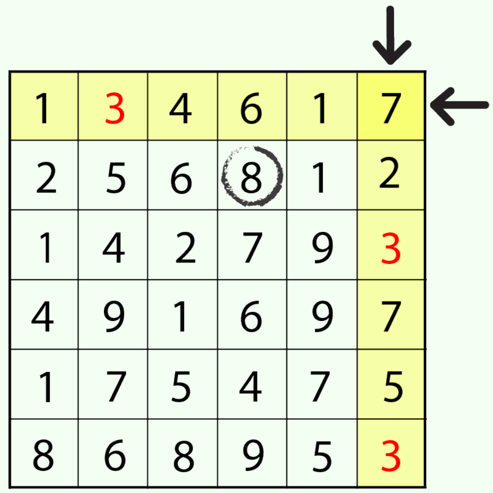
In this example, the number 3 is only found in the squares of the highlighted row. Since the sum of numbers in this row is at least 3, we can cross out 7 and 9 since $3 + 7 = 10$ and $3 + 9 = 12$, and both 10 and 12 are greater than 9. Equivalently, the value of $9 - n$ in this case is $9 - 3 = 6$ so we know to cross out all the numbers that are greater than 6.

2. After shading in any series of numbers, check if you eliminated enough to narrow down exactly one of any number.

Using our above example, by crossing out the 7 in the third row, there is only one box left in the whole grid that has the number 7, so we can go ahead and circle it. Our next step would be to cross out all of the numbers in the row and column of this box that are greater than 2 ($9 - n = 9 - 7 = 2$):



3. Similarly to the first strategy, try to identify numbers that exist in exactly one row **and** exactly one column, and see if you can eliminate the box they share. We will see an example of this below!



In this grid, the number 3 can be found in exactly one row, and exactly one column. This row and column *intersect* in exactly one square in the top right corner (which holds the number 7). We can go ahead and cross out this number 7 since you cannot circle the number 3 without it being in the same row/column as this box, and the sum of 3 and 7 is greater than 9 which breaks the rules of the puzzle.

4. If you are completely stuck, use trial and error. Pick one of the bigger numbers you have left and choose a box to circle; see if you can solve the puzzle, and if you come to a problem, that means that was the wrong box to circle!

Unfortunately we have reached the end of our lesson, but do not fret! The *logical* next step is to work on the problem set!